# Deriving Pretty-Big-Step Semantics from Small-Step Semantics

Casper Bach Poulsen and Peter D. Mosses

Department of Computer Science
Swansea University
Swansea, UK

{cscbp,p.d.mosses}@swansea.ac.uk

9 April
ESOP 2014, Grenoble, France

# Small-step or big-step semantics?

For expressions $e \in \mathit{Expr}$ and values $v \in \mathit{Val}$

- Small-step – relates partly evaluated expressions $\boxed{e \to e'}$
- Big-step – relates expressions to final values $\boxed{e \Rightarrow v}$

# Small-step vs. big-step SOS

## Language syntax

$$Expr \ni e ::= v \mid t$$

$$Val \ni v ::= n \in \mathbb{N} \qquad Term \ni t ::= \mathsf{plus}(e, e)$$

# Small-step vs. big-step SOS

## Language syntax

$$Expr \ni e ::= v \mid t$$

$$Val \ni v ::= n \in \mathbb{N} \qquad Term \ni t ::= \mathsf{plus}(e, e)$$

# Small-step vs. big-step SOS

## Language syntax

$$Expr \ni e ::= v \mid t$$

$$\boxed{Val \ni v ::= n \in \mathbb{N}} \qquad Term \ni t ::= \mathsf{plus}(e, e)$$

# Small-step vs. big-step SOS

## Language syntax

$$Expr \ni e ::= v \mid t$$

$$Val \ni v ::= n \in \mathbb{N} \qquad Term \ni t ::= \mathsf{plus}(e, e)$$

# Small-step vs. big-step SOS

## Language syntax

$$Expr \ni e ::= v \mid t$$

$$Val \ni v ::= n \in \mathbb{N} \qquad Term \ni t ::= \mathsf{plus}(e, e)$$

## Small-step rules

$$\boxed{e \to e'}$$

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \to n} \text{[PLUS]}$$

$$\frac{t_1 \to e_1'}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(e_1', e_2)} \text{[PLUS1]} \qquad \frac{t_2 \to e_2'}{\mathsf{plus}(v_1, t_2) \to \mathsf{plus}(v_1, e_2')} \text{[PLUS2]}$$

# Small-step vs. big-step SOS

## Language syntax

$$Expr \ni e ::= v \mid t$$

$$Val \ni v ::= n \in \mathbb{N} \qquad Term \ni t ::= \mathsf{plus}(e, e)$$

## Small-step rules

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \to n} \text{ [PLUS]} \qquad \boxed{e \to e'}$$

$$\frac{t_1 \to e_1'}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(e_1', e_2)} \text{ [PLUS1]} \qquad \frac{t_2 \to e_2'}{\mathsf{plus}(v_1, t_2) \to \mathsf{plus}(v_1, e_2')} \text{ [PLUS2]}$$

# Small-step vs. big-step SOS

## Language syntax

$$Expr \ni e ::= v \mid t$$

$$Val \ni v ::= n \in \mathbb{N} \qquad Term \ni t ::= \mathsf{plus}(e, e)$$

## Small-step rules

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \to n} \text{[PLUS]} \qquad \boxed{e \to e'}$$

$$\frac{t_1 \to e_1'}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(e_1', e_2)} \text{[PLUS1]} \qquad \frac{t_2 \to e_2'}{\mathsf{plus}(v_1, t_2) \to \mathsf{plus}(v_1, e_2')} \text{[PLUS2]}$$

# Small-step vs. big-step SOS

## Language syntax

$$Expr \ni e ::= v \mid t$$

$$Val \ni v ::= n \in \mathbb{N} \qquad Term \ni t ::= \mathsf{plus}(e, e)$$

## Small-step rules

$$\boxed{e \to e'}$$

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \to n} \text{[PLUS]}$$

$$\frac{t_1 \to e_1'}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(e_1', e_2)} \text{[PLUS1]} \qquad \frac{t_2 \to e_2'}{\mathsf{plus}(v_1, t_2) \to \mathsf{plus}(v_1, e_2')} \text{[PLUS2]}$$

# Small-step vs. big-step SOS

## Language syntax

$$Expr \ni e ::= v \mid t$$

$$Val \ni v ::= n \in \mathbb{N} \qquad Term \ni t ::= \mathsf{plus}(e, e)$$

## Small-step rules

$$\boxed{e \to e'}$$

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \to n} \text{[PLUS]}$$

$$\frac{t_1 \to e_1'}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(e_1', e_2)} \text{[PLUS1]} \qquad \frac{t_2 \to e_2'}{\mathsf{plus}(v_1, t_2) \to \mathsf{plus}(v_1, e_2')} \text{[PLUS2]}$$

# Small-step vs. big-step SOS

## Language syntax

$$Expr \ni e ::= v \mid t$$

$$Val \ni v ::= n \in \mathbb{N} \qquad Term \ni t ::= \mathsf{plus}(e, e)$$

## Small-step rules

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \to n}\ [\textsc{Plus}] \qquad \boxed{e \to e'}$$

$$\frac{t_1 \to e_1'}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(e_1', e_2)}\ [\textsc{Plus1}] \qquad \frac{t_2 \to e_2'}{\mathsf{plus}(v_1, t_2) \to \mathsf{plus}(v_1, e_2')}\ [\textsc{Plus2}]$$

## Big-step rules

$$\frac{}{v \Rightarrow v}\ [\textsc{ReflV}] \qquad \frac{e_1 \Rightarrow n_1 \qquad e_2 \Rightarrow n_2 \qquad n = n_1 + n_2}{\mathsf{plus}(e_1, e_2) \Rightarrow n}\ [\textsc{BPlus}] \qquad \boxed{e \Rightarrow v}$$

# Small-step vs. big-step SOS

## Language syntax

$$Expr \ni e ::= v \mid t$$

$$Val \ni v ::= n \in \mathbb{N} \qquad Term \ni t ::= \mathsf{plus}(e, e)$$

## Small-step rules

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \rightarrow n}\ [\textsc{Plus}] \qquad \boxed{e \rightarrow e'}$$

$$\frac{t_1 \rightarrow e_1'}{\mathsf{plus}(t_1, e_2) \rightarrow \mathsf{plus}(e_1', e_2)}\ [\textsc{Plus1}] \qquad \frac{t_2 \rightarrow e_2'}{\mathsf{plus}(v_1, t_2) \rightarrow \mathsf{plus}(v_1, e_2')}\ [\textsc{Plus2}]$$

## Big-step rules

$$\boxed{v \Rightarrow v}\ [\textsc{ReflV}] \qquad \frac{e_1 \Rightarrow n_1 \qquad e_2 \Rightarrow n_2 \qquad n = n_1 + n_2}{\mathsf{plus}(e_1, e_2) \Rightarrow n}\ [\textsc{BPlus}] \qquad \boxed{e \Rightarrow v}$$

# Small-step vs. big-step SOS

## Language syntax

$$Expr \ni e ::= v \mid t$$

$$Val \ni v ::= n \in \mathbb{N} \qquad Term \ni t ::= \mathsf{plus}(e, e)$$

## Small-step rules

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \to n} \text{ [PLUS]} \qquad \boxed{e \to e'}$$

$$\frac{t_1 \to e_1'}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(e_1', e_2)} \text{ [PLUS1]} \qquad \frac{t_2 \to e_2'}{\mathsf{plus}(v_1, t_2) \to \mathsf{plus}(v_1, e_2')} \text{ [PLUS2]}$$

## Big-step rules

$$\frac{}{v \Rightarrow v} \text{ [REFLV]} \qquad \frac{e_1 \Rightarrow n_1 \qquad e_2 \Rightarrow n_2 \qquad n = n_1 + n_2}{\mathsf{plus}(e_1, e_2) \Rightarrow n} \text{ [BPLUS]} \qquad \boxed{e \Rightarrow v}$$

# Small-step vs. big-step SOS

## Language syntax

$$Expr \ni e ::= v \mid t$$

$$Val \ni v ::= n \in \mathbb{N} \qquad Term \ni t ::= \mathsf{plus}(e, e)$$

## Small-step rules

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \to n}\ [\textsc{Plus}] \qquad \boxed{e \to e'}$$

$$\frac{t_1 \to e_1'}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(e_1', e_2)}\ [\textsc{Plus1}] \qquad \frac{t_2 \to e_2'}{\mathsf{plus}(v_1, t_2) \to \mathsf{plus}(v_1, e_2')}\ [\textsc{Plus2}]$$

## Big-step rules

$$\frac{}{v \Rightarrow v}\ [\textsc{ReflV}] \qquad \frac{e_1 \Rightarrow n_1 \qquad \boxed{e_2 \Rightarrow n_2} \qquad n = n_1 + n_2}{\mathsf{plus}(e_1, e_2) \Rightarrow n}\ [\textsc{BPlus}] \qquad \boxed{e \Rightarrow v}$$

# Small-step vs. big-step SOS

## Language syntax

$$Expr \ni e ::= v \mid t$$

$$Val \ni v ::= n \in \mathbb{N} \qquad Term \ni t ::= \mathsf{plus}(e, e)$$

## Small-step rules

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \to n}\ [\text{PLUS}] \qquad \boxed{e \to e'}$$

$$\frac{t_1 \to e_1'}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(e_1', e_2)}\ [\text{PLUS1}] \qquad \frac{t_2 \to e_2'}{\mathsf{plus}(v_1, t_2) \to \mathsf{plus}(v_1, e_2')}\ [\text{PLUS2}]$$

## Big-step rules

$$\frac{}{v \Rightarrow v}\ [\text{REFLV}] \qquad \frac{e_1 \Rightarrow n_1 \qquad e_2 \Rightarrow n_2 \qquad n = n_1 + n_2}{\mathsf{plus}(e_1, e_2) \Rightarrow n}\ [\text{BPLUS}] \qquad \boxed{e \Rightarrow v}$$

# Small-step vs. big-step SOS

## Language syntax

$$Expr \ni e ::= v \mid t$$

$$Val \ni v ::= n \in \mathbb{N} \qquad Term \ni t ::= \mathsf{plus}(e, e)$$

## Small-step rules

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \to n} \text{[PLUS]} \qquad \boxed{e \to e'}$$

$$\frac{t_1 \to e_1'}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(e_1', e_2)} \text{[PLUS1]} \qquad \frac{t_2 \to e_2'}{\mathsf{plus}(v_1, t_2) \to \mathsf{plus}(v_1, e_2')} \text{[PLUS2]}$$

## Big-step rules

$$\frac{}{v \Rightarrow v} \text{[REFLV]} \qquad \frac{e_1 \Rightarrow n_1 \qquad e_2 \Rightarrow n_2 \qquad n = n_1 + n_2}{\mathsf{plus}(e_1, e_2) \Rightarrow n} \text{[BPLUS]} \qquad \boxed{e \Rightarrow v}$$

# Which style to use?

Big-step:

- duplication problem in the presence of abrupt termination [Charguéraud, ESOP'13]

# Which style to use?

Big-step:

- duplication problem in the presence of abrupt termination [Charguéraud, ESOP'13]

**Big-step with abrupt termination**

$$\frac{}{v \Rightarrow v}\ [\textsc{ReflV}] \qquad \frac{}{\mathsf{throw} \Rightarrow \mathsf{exc}}\ [\textsc{BThrow}] \qquad \boxed{e \Rightarrow o}$$

$$\frac{e_1 \Rightarrow \mathsf{exc}}{\mathsf{plus}(e_1, e_2) \Rightarrow \mathsf{exc}}\ [\textsc{BPlus1-Exc}]$$

$$\frac{e_1 \Rightarrow n_1 \qquad e_2 \Rightarrow \mathsf{exc}}{\mathsf{plus}(e_1, e_2) \Rightarrow \mathsf{exc}}\ [\textsc{BPlus2-exc}]$$

$$\frac{e_1 \Rightarrow n_1 \qquad e_2 \Rightarrow n_2 \qquad n = n_1 + n_2}{\mathsf{plus}(e_1, e_2) \Rightarrow n}\ [\textsc{BPlus}]$$

# Which style to use?

Big-step:

- duplication problem in the presence of abrupt termination [Charguéraud, ESOP'13]

**Big-step with abrupt termination**

$$\frac{}{v \Rightarrow v}\ [\textsc{ReflV}] \qquad \frac{}{\mathsf{throw} \Rightarrow \mathsf{exc}}\ [\textsc{BThrow}] \qquad \boxed{e \Rightarrow o}$$

$$\frac{e_1 \Rightarrow \mathsf{exc}}{\mathsf{plus}(e_1, e_2) \Rightarrow \mathsf{exc}}\ [\textsc{BPlus1-Exc}]$$

$$\frac{e_1 \Rightarrow n_1 \qquad e_2 \Rightarrow \mathsf{exc}}{\mathsf{plus}(e_1, e_2) \Rightarrow \mathsf{exc}}\ [\textsc{BPlus2-exc}]$$

$$\frac{e_1 \Rightarrow n_1 \qquad e_2 \Rightarrow n_2 \qquad n = n_1 + n_2}{\mathsf{plus}(e_1, e_2) \Rightarrow n}\ [\textsc{BPlus}]$$

# Which style to use?

Big-step:

- duplication problem in the presence of abrupt termination [Charguéraud, ESOP'13]

**Big-step with abrupt termination**

$$\frac{}{v \Rightarrow v}\ [\textsc{ReflV}] \qquad \frac{}{\mathsf{throw} \Rightarrow \mathsf{exc}}\ [\textsc{BThrow}] \qquad \boxed{e \Rightarrow o}$$

$$\frac{e_1 \Rightarrow \mathsf{exc}}{\mathsf{plus}(e_1, e_2) \Rightarrow \mathsf{exc}}\ [\textsc{BPlus1-Exc}]$$

$$\frac{\boxed{e_1 \Rightarrow n_1} \qquad e_2 \Rightarrow \mathsf{exc}}{\mathsf{plus}(e_1, e_2) \Rightarrow \mathsf{exc}}\ [\textsc{BPlus2-exc}]$$

$$\frac{\boxed{e_1 \Rightarrow n_1} \qquad e_2 \Rightarrow n_2 \qquad n = n_1 + n_2}{\mathsf{plus}(e_1, e_2) \Rightarrow n}\ [\textsc{BPlus}]$$

# Which style to use?

Big-step:

- duplication problem in the presence of abrupt termination [Charguéraud, ESOP'13]

Pretty-big-step [Charguéraud, ESOP'13]:

- solves the duplication problem for abrupt termination

# Which style to use?

Big-step:

- duplication problem in the presence of abrupt termination

**Pretty-big-step with abrupt termination**

$$\frac{}{v \Downarrow v}\ [\textsc{ReflV}] \qquad \frac{}{\mathsf{throw} \Downarrow \mathsf{exc}}\ [\textsc{PBThrow}] \qquad \boxed{e \Downarrow o}$$

$$\frac{t_1 \Downarrow o_1 \qquad \mathsf{plus}(o_1, e_2) \Downarrow o}{\mathsf{plus}(t_1, e_2) \Downarrow o}\ [\textsc{PBPlus1}]$$

$$\frac{t_2 \Downarrow o_2 \qquad \mathsf{plus}(v_1, o_2) \Downarrow o}{\mathsf{plus}(v_1, t_2) \Downarrow o}\ [\textsc{PBPlus2}]$$

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \Downarrow n}\ [\textsc{PBPlus}]$$

$$\frac{}{\mathsf{plus}(\mathsf{exc}, e_2) \Downarrow \mathsf{exc}}\ [\textsc{PBPlus1-Exc}] \qquad \frac{}{\mathsf{plus}(v_1, \mathsf{exc}) \Downarrow \mathsf{exc}}\ [\textsc{PBPlus2-Exc}]$$

# Which style to use?

Big-step:

- duplication problem in the presence of abrupt termination

**Pretty-big-step with abrupt termination**

$e \Downarrow o$

$$\frac{}{v \Downarrow v}\ [\textsc{ReflV}] \qquad \frac{}{\mathsf{throw} \Downarrow \mathsf{exc}}\ [\textsc{PBThrow}]$$

$$\frac{t_1 \Downarrow o_1 \qquad \mathsf{plus}(o_1, e_2) \Downarrow o}{\mathsf{plus}(t_1, e_2) \Downarrow o}\ [\textsc{PBPlus1}]$$

$$\frac{t_2 \Downarrow o_2 \qquad \mathsf{plus}(v_1, o_2) \Downarrow o}{\mathsf{plus}(v_1, t_2) \Downarrow o}\ [\textsc{PBPlus2}]$$

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \Downarrow n}\ [\textsc{PBPlus}]$$

$$\frac{}{\mathsf{plus}(\mathsf{exc}, e_2) \Downarrow \mathsf{exc}}\ [\textsc{PBPlus1-Exc}] \qquad \frac{}{\mathsf{plus}(v_1, \mathsf{exc}) \Downarrow \mathsf{exc}}\ [\textsc{PBPlus2-Exc}]$$

# Which style to use?

Big-step:

- duplication problem in the presence of abrupt termination

**Pretty-big-step with abrupt termination**

$$\frac{}{v \Downarrow v}\ [\text{ReflV}] \qquad \frac{}{\mathsf{throw} \Downarrow \mathsf{exc}}\ [\text{PBThrow}] \qquad \boxed{e \Downarrow o}$$

$$\frac{t_1 \Downarrow o_1 \qquad \mathsf{plus}(o_1, e_2) \Downarrow o}{\mathsf{plus}(t_1, e_2) \Downarrow o}\ [\text{PBPlus1}]$$

$$\frac{t_2 \Downarrow o_2 \qquad \mathsf{plus}(v_1, o_2) \Downarrow o}{\mathsf{plus}(v_1, t_2) \Downarrow o}\ [\text{PBPlus2}]$$

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \Downarrow n}\ [\text{PBPlus}]$$

$$\frac{}{\mathsf{plus}(\mathsf{exc}, e_2) \Downarrow \mathsf{exc}}\ [\text{PBPlus1-Exc}] \qquad \frac{}{\mathsf{plus}(v_1, \mathsf{exc}) \Downarrow \mathsf{exc}}\ [\text{PBPlus2-Exc}]$$

# Which style to use?

Big-step:

- duplication problem in the presence of abrupt termination

## Pretty-big-step with abrupt termination

$$\frac{}{v \Downarrow v}\ [\text{ReflV}] \qquad \frac{}{\mathsf{throw} \Downarrow \mathsf{exc}}\ [\text{PBThrow}] \qquad \boxed{e \Downarrow o}$$

$$\frac{t_1 \Downarrow o_1 \qquad \mathsf{plus}(o_1, e_2) \Downarrow o}{\mathsf{plus}(t_1, e_2) \Downarrow o}\ [\text{PBPlus1}]$$

$$\frac{t_2 \Downarrow o_2 \qquad \mathsf{plus}(v_1, o_2) \Downarrow o}{\mathsf{plus}(v_1, t_2) \Downarrow o}\ [\text{PBPlus2}]$$

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \Downarrow n}\ [\text{PBPlus}]$$

$$\frac{}{\mathsf{plus}(\mathsf{exc}, e_2) \Downarrow \mathsf{exc}}\ [\text{PBPlus1-Exc}] \qquad \frac{}{\mathsf{plus}(v_1, \mathsf{exc}) \Downarrow \mathsf{exc}}\ [\text{PBPlus2-Exc}]$$

# Which style to use?

Big-step:

- duplication problem in the presence of abrupt termination

**Pretty-big-step with abrupt termination**

$$\frac{}{v \Downarrow v}\ [\textsc{ReflV}] \qquad \frac{}{\mathsf{throw} \Downarrow \mathsf{exc}}\ [\textsc{PBThrow}] \qquad \boxed{e \Downarrow o}$$

$$\frac{t_1 \Downarrow o_1 \qquad \mathsf{plus}(o_1, e_2) \Downarrow o}{\mathsf{plus}(t_1, e_2) \Downarrow o}\ [\textsc{PBPlus1}]$$

$$\frac{t_2 \Downarrow o_2 \qquad \mathsf{plus}(v_1, o_2) \Downarrow o}{\mathsf{plus}(v_1, t_2) \Downarrow o}\ [\textsc{PBPlus2}]$$

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \Downarrow n}\ [\textsc{PBPlus}]$$

$$\frac{}{\mathsf{plus}(\mathsf{exc}, e_2) \Downarrow \mathsf{exc}}\ [\textsc{PBPlus1-Exc}] \qquad \frac{}{\mathsf{plus}(v_1, \mathsf{exc}) \Downarrow \mathsf{exc}}\ [\textsc{PBPlus2-Exc}]$$

# Which style to use?

Big-step:

- duplication problem in the presence of abrupt termination [Charguéraud, ESOP'13]

Pretty-big-step [Charguéraud, ESOP'13]:

- solves the duplication problem for abrupt termination
- pretty-big-step reasoning (compiler correctness, efficient interpreters), similar to traditional big-step

# Which style to use?

Big-step:

- duplication problem in the presence of abrupt termination [Charguéraud, ESOP'13]

Pretty-big-step [Charguéraud, ESOP'13]:

- solves the duplication problem for abrupt termination
- pretty-big-step reasoning (compiler correctness, efficient interpreters), similar to traditional big-step
- but less convenient for small-step reasoning (type soundness, interleaving)

# Which style to use?

Big-step:

- duplication problem in the presence of abrupt termination [Charguéraud, ESOP'13]

Pretty-big-step [Charguéraud, ESOP'13]:

- solves the duplication problem for abrupt termination
- pretty-big-step reasoning (compiler correctness, efficient interpreters), similar to traditional big-step
- but less convenient for small-step reasoning (type soundness, interleaving)

Small-step:

- no duplication problem for abrupt termination

# Which style to use?

Big-step:

## Language syntax

$$Expr \ni e ::= v \mid t \quad Val \ni v ::= n \in \mathbb{N} \quad Term \ni t ::= \mathsf{plus}(e, e)$$

$$AbrTer \ni a ::= \tau \mid \mathsf{exc} \qquad Outcome \ni o ::= \langle v, \tau \rangle \mid \langle e, \mathsf{exc} \rangle$$

- solves the duplication problem for abrupt termination

## Small-step with abrupt termination

$$\langle e, a \rangle \to \langle e', a' \rangle$$

$$\frac{}{\langle \mathsf{throw}, \tau \rangle \to \langle 0, \mathsf{exc} \rangle} \text{[THROW]}$$

$$\frac{n = n_1 + n_2}{\langle \mathsf{plus}(n_1, n_2), \tau \rangle \to \langle n, \tau \rangle} \text{[PLUS]}$$

$$\frac{\langle t_1, \tau \rangle \to \langle e'_1, a' \rangle}{\langle \mathsf{plus}(t_1, e_2), \tau \rangle \to \langle \mathsf{plus}(e'_1, e_2), a' \rangle} \text{[PLUS1]}$$

$$\frac{\langle t_2, \tau \rangle \to \langle e'_2, a' \rangle}{\langle \mathsf{plus}(v_1, t_2), \tau \rangle \to \langle \mathsf{plus}(v_1, e'_2), a' \rangle} \text{[PLUS2]}$$

# Which style to use?

Big-step:

## Language syntax

$$Expr \ni e ::= v \mid t \quad Val \ni v ::= n \in \mathbb{N} \quad Term \ni t ::= \mathsf{plus}(e, e)$$

$$AbrTer \ni a ::= \tau \mid \mathsf{exc} \qquad Outcome \ni o ::= \langle v, \tau \rangle \mid \langle e, \mathsf{exc} \rangle$$

- solves the duplication problem for abrupt termination

## Small-step with abrupt termination

$\langle e, a \rangle \to \langle e', a' \rangle$

$$\frac{}{\langle \mathsf{throw}, \tau \rangle \to \langle 0, \mathsf{exc} \rangle} \text{[THROW]}$$

$$\frac{n = n_1 + n_2}{\langle \mathsf{plus}(n_1, n_2), \tau \rangle \to \langle n, \tau \rangle} \text{[PLUS]}$$

$$\frac{\langle t_1, \tau \rangle \to \langle e_1', a' \rangle}{\langle \mathsf{plus}(t_1, e_2), \tau \rangle \to \langle \mathsf{plus}(e_1', e_2), a' \rangle} \text{[PLUS1]}$$

$$\frac{\langle t_2, \tau \rangle \to \langle e_2', a' \rangle}{\langle \mathsf{plus}(v_1, t_2), \tau \rangle \to \langle \mathsf{plus}(v_1, e_2'), a' \rangle} \text{[PLUS2]}$$

# Which style to use?

Big-step:

**Language syntax**

$$Expr \ni e ::= v \mid t \quad Val \ni v ::= n \in \mathbb{N} \quad Term \ni t ::= \mathsf{plus}(e, e)$$

$$AbrTer \ni a ::= \tau \mid \mathsf{exc} \qquad Outcome \ni o ::= \langle v, \tau \rangle \mid \langle e, \mathsf{exc} \rangle$$

- solves the duplication problem for abrupt termination

**Small-step with abrupt termination**

$$\langle e, a \rangle \to \langle e', a' \rangle$$

$$\frac{}{\langle \mathsf{throw}, \tau \rangle \to \langle 0, \mathsf{exc} \rangle}\ [\textsc{Throw}]$$

$$\frac{n = n_1 + n_2}{\langle \mathsf{plus}(n_1, n_2), \tau \rangle \to \langle n, \tau \rangle}\ [\textsc{Plus}]$$

$$\frac{\langle t_1, \tau \rangle \to \langle e'_1, a' \rangle}{\langle \mathsf{plus}(t_1, e_2), \tau \rangle \to \langle \mathsf{plus}(e'_1, e_2), a' \rangle}\ [\textsc{Plus1}]$$

$$\frac{\langle t_2, \tau \rangle \to \langle e'_2, a' \rangle}{\langle \mathsf{plus}(v_1, t_2), \tau \rangle \to \langle \mathsf{plus}(v_1, e'_2), a' \rangle}\ [\textsc{Plus2}]$$

# Which style to use?

Big-step:

- duplication problem in the presence of abrupt termination [Charguéraud, ESOP'13]

Pretty-big-step [Charguéraud, ESOP'13]:

- solves the duplication problem for abrupt termination
- pretty-big-step reasoning (compiler correctness, efficient interpreters), similar to traditional big-step
- but less convenient for small-step reasoning (type soundness, interleaving)

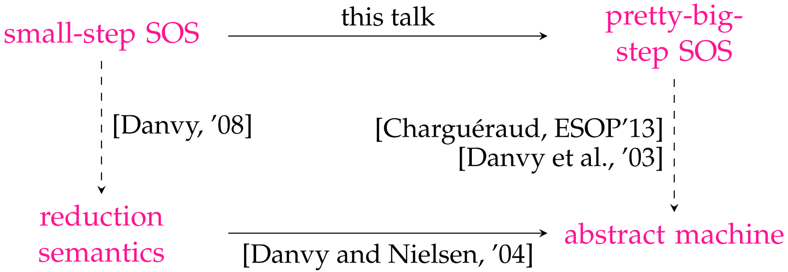
Small-step:

- no duplication problem for abrupt termination
- but less convenient for big-step reasoning (compiler correctness, efficient interpreters)

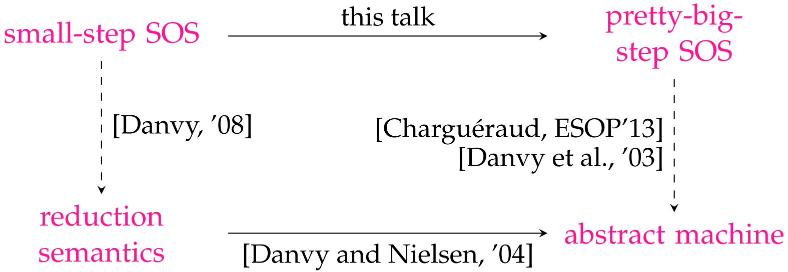
# We don't have to choose!

small-step SOS $\xrightarrow{\text{this talk}}$ pretty-big-step SOS

# We don't have to choose!



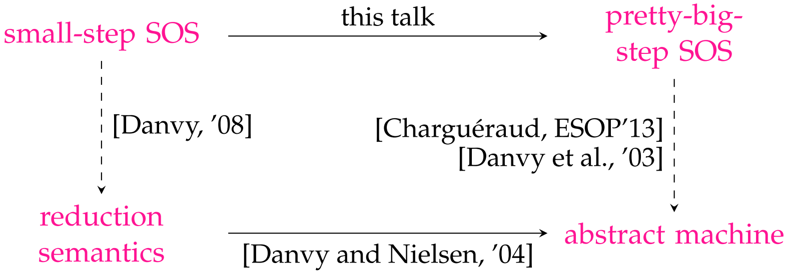
# We don't have to choose!




Talk overview:

- refocusing: derivation producing pretty-big-step semantics from small-step semantics

# We don't have to choose!




Talk overview:

- refocusing: derivation producing pretty-big-step semantics from small-step semantics
- correctness: proof method and Coq mechanisation

# Refocusing

from small-step to pretty-big-step evaluation

# Small-step SOS evaluation

Evaluation rules

$\boxed{e \rightarrow e'}$ $\boxed{e \searrow v}$

$$\frac{t \rightarrow e' \qquad e' \searrow v}{t \searrow v}\ [\textsc{Trans}] \qquad \frac{}{v \searrow v}\ [\textsc{ReflV}]$$

# Small-step SOS evaluation

## Evaluation rules

$\boxed{e \to e'}$ $\boxed{e \searrow v}$

$$\frac{t \to e' \qquad e' \searrow v}{t \searrow v}\ [\textsc{Trans}] \qquad\qquad \frac{}{v \searrow v}\ [\textsc{ReflV}]$$

## Derivation tree structure

$$\frac{\dfrac{\vdots}{A} \qquad \dfrac{\dfrac{\vdots}{B} \qquad \dfrac{\vdots \qquad \iddots}{\Psi}\ [\textsc{Trans}]}{\Delta}\ [\textsc{Trans}]}{\Gamma}\ [\textsc{Trans}]$$

# Refocusing in SOS

Small-step $\longrightarrow$ pretty-big-step SOS by

1. introduce refocusing rule
2. specialize congruence rules
3. specialize evaluation rules

# Refocusing in SOS

Small-step $\longrightarrow$ pretty-big-step SOS by

1. introduce refocusing rule
2. specialize congruence rules
3. specialize evaluation rules

Fully automatic: prototyped in Prolog, used to generate prototype interpreters

# Refocusing in SOS

Small-step $\longrightarrow$ pretty-big-step SOS by

1. introduce refocusing rule
2. specialize congruence rules
3. specialize evaluation rules

Fully automatic: prototyped in Prolog, used to generate prototype interpreters

# 1. Introduce refocusing rule

Evaluation rules

$\boxed{e \rightarrow e'}$ $\boxed{e \searrow n}$

$$\frac{t \rightarrow e' \qquad e' \searrow v}{t \searrow v}\ [\textsc{Trans}] \qquad \frac{}{v \searrow v}\ [\textsc{ReflV}]$$

Idea: allow big-steps anywhere

$$\frac{t \searrow v}{t \rightarrow v}\ [\textsc{Refocus}]$$

# 1. Introduce refocusing rule

Evaluation rules $\boxed{e \to e'}$ $\boxed{e \searrow n}$

$$\frac{t \to e' \qquad e' \searrow v}{t \searrow v}\ [\textsc{Trans}] \qquad \frac{}{v \searrow v}\ [\textsc{ReflV}]$$

Idea: allow big-steps anywhere

$$\frac{t \searrow v}{\boxed{t \to v}}\ [\textsc{Refocus}]$$

# 1. Introduce refocusing rule

Evaluation rules

$\boxed{e \to e'}$ $\boxed{e \searrow n}$

$$\frac{t \to e' \qquad e' \searrow v}{t \searrow v}\ [\textsc{Trans}] \qquad \frac{}{v \searrow v}\ [\textsc{ReflV}]$$

Idea: allow big-steps anywhere

$$\frac{\boxed{t \searrow v}}{t \to v}\ [\textsc{Refocus}]$$

# 1. Introduce refocusing rule

## Evaluation rules

$\boxed{e \rightarrow e'}$ $\boxed{e \searrow n}$

$$\frac{t \rightarrow e' \qquad e' \searrow v}{t \searrow v}\,[\textsc{Trans}] \qquad\qquad \frac{}{v \searrow v}\,[\textsc{ReflV}]$$

## Idea: allow big-steps anywhere

$$\frac{t \searrow v}{t \rightarrow v}\,[\textsc{Refocus}]$$

## Derivation tree structure

$$\dfrac{[r]\,\dfrac{[\textsc{Refocus}]\,\dfrac{[\textsc{Trans}]\,\dfrac{\dfrac{\vdots}{C} \qquad \dfrac{\vdots \qquad \iddots}{\Phi}}{\Psi}}{B}}{A} \qquad\qquad \Delta}{\Gamma}\,[\textsc{Trans}]$$

# 1. Introduce refocusing rule

## Evaluation rules

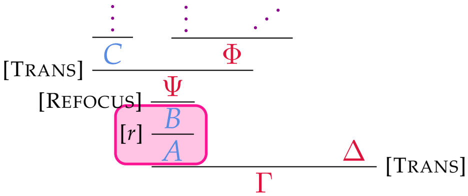
$\boxed{e \rightarrow e'}$ $\boxed{e \searrow n}$

$$\frac{t \rightarrow e' \qquad e' \searrow v}{t \searrow v}\ [\textsc{Trans}] \qquad\qquad \frac{}{v \searrow v}\ [\textsc{ReflV}]$$

## Idea: allow big-steps anywhere

$$\frac{t \searrow v}{t \rightarrow v}\ [\textsc{Refocus}]$$

## Derivation tree structure



# 1. Introduced refocusing rule

## Evaluation rules

$$\frac{t \rightarrow e' \qquad e' \searrow v}{t \searrow v}\,[\textsc{Trans}] \qquad \frac{}{v \searrow v}\,[\textsc{ReflV}] \qquad \frac{t \searrow v}{t \rightarrow v}\,[\textsc{Refocus}]$$

Potential issues?

# 1. Introduced refocusing rule

## Evaluation rules

$$\frac{t \to e' \qquad e' \searrow v}{t \searrow v}\ [\textsc{Trans}] \qquad \frac{}{v \searrow v}\ [\textsc{ReflV}] \qquad \frac{t \searrow v}{t \to v}\ [\textsc{Refocus}]$$

Potential issues?

- mutually inductive $\to$ and $\searrow$

# 1. Introduced refocusing rule

**Evaluation rules**

$$\frac{t \rightarrow e' \qquad e' \searrow v}{t \searrow v}\ [\textsc{Trans}] \qquad \frac{}{v \searrow v}\ [\textsc{ReflV}] \qquad \frac{t \searrow v}{t \rightarrow v}\ [\textsc{Refocus}]$$

Potential issues?

- mutually inductive $\rightarrow$ and $\searrow$
- nondeterministic choice between ordinary and refocused steps

# 1. Introduced refocusing rule

## Evaluation rules

$$\frac{t \rightarrow e' \qquad e' \searrow v}{t \searrow v}\,[\textsc{Trans}] \qquad \frac{}{v \searrow v}\,[\textsc{ReflV}] \qquad \frac{t \searrow v}{t \rightarrow v}\,[\textsc{Refocus}]$$

Potential issues?

- mutually inductive $\rightarrow$ and $\searrow$
- nondeterministic choice between ordinary and refocused steps
- non-termination for left-recursive interpretation

$$\dfrac{\dfrac{\dfrac{\vdots}{t \searrow v}\,[\textsc{Trans}]}{t \rightarrow v}\,[\textsc{Refocus}] \qquad v \searrow v}{t \searrow v}\,[\textsc{Trans}]$$

# Refocusing in SOS

Small-step $\longrightarrow$ pretty-big-step SOS by

1. introduce refocusing rule
2. specialize congruence rules
3. specialize evaluation rules

# 2. Specialize congruence rules

Congruence rules

$$\frac{t_1 \rightarrow e_1'}{\mathsf{plus}(t_1, e_2) \rightarrow \mathsf{plus}(e_1', e_2)} \text{ [PLUS1]} \qquad \frac{t_2 \rightarrow e_2'}{\mathsf{plus}(v_1, t_2) \rightarrow \mathsf{plus}(v_1, e_2')} \text{ [PLUS2]}$$

# 2. Specialize congruence rules

Congruence rules

$$\frac{t_1 \to e_1'}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(e_1', e_2)}\ [\textsc{Plus1}] \qquad \frac{t_2 \to e_2'}{\mathsf{plus}(v_1, t_2) \to \mathsf{plus}(v_1, e_2')}\ [\textsc{Plus2}]$$

Specialization for full evaluation of subterms:

$$\frac{t_1 \to e_1'}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(e_1', e_2)}\ [\textsc{Plus1}]$$

# 2. Specialize congruence rules

Congruence rules

$$\frac{t_1 \to e_1'}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(e_1', e_2)}\ [\text{PLUS1}] \qquad \frac{t_2 \to e_2'}{\mathsf{plus}(v_1, t_2) \to \mathsf{plus}(v_1, e_2')}\ [\text{PLUS2}]$$

Specialization for full evaluation of subterms:

$$\frac{\dfrac{t_1 \searrow v_1}{t_1 \to v_1}\ [\text{REFOCUS}]}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(v_1, e_2)}\ [\text{PLUS1}]$$

# 2. Specialize congruence rules

Congruence rules

$$\frac{t_1 \to e_1'}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(e_1', e_2)}\ [\textsc{Plus1}] \qquad \frac{t_2 \to e_2'}{\mathsf{plus}(v_1, t_2) \to \mathsf{plus}(v_1, e_2')}\ [\textsc{Plus2}]$$

Specialization for full evaluation of subterms:

$$\frac{t_1 \searrow v_1}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(v_1, e_2)}\ [\textsc{Plus1}']$$

# 2. Specialize congruence rules

Congruence rules

$$\frac{t_1 \to e_1'}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(e_1', e_2)}\ [\textsc{Plus1}] \qquad \frac{t_2 \to e_2'}{\mathsf{plus}(v_1, t_2) \to \mathsf{plus}(v_1, e_2')}\ [\textsc{Plus2}]$$

Specialization for full evaluation of subterms:

$$\frac{t_1 \searrow v_1}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(v_1, e_2)}\ [\textsc{Plus1}'] \qquad \frac{t_2 \searrow v_2}{\mathsf{plus}(v_1, t_2) \to \mathsf{plus}(v_1, v_2)}\ [\textsc{Plus2}']$$

# 2. Specialized congruence rules

## Language syntax

$$Expr \ni e ::= v \mid t$$

$$Val \ni v ::= n \in \mathbb{N} \qquad Term \ni t ::= \mathsf{plus}(e, e)$$

## Rules

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \to n}\ [\textsc{Plus}]$$

$$\frac{t_1 \searrow v_1}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(v_1, e_2)}\ [\textsc{Plus1}'] \qquad \frac{t_2 \searrow v_2}{\mathsf{plus}(v_1, t_2) \to \mathsf{plus}(v_1, v_2)}\ [\textsc{Plus2}']$$

## Evaluation rules

$$\frac{t' \to e' \qquad e \searrow v}{t \searrow v}\ [\textsc{Trans}] \qquad \frac{}{v \searrow v}\ [\textsc{ReflV}]$$

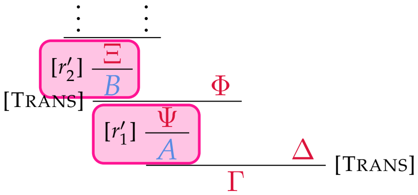
# 2. Specialized congruence rules

Subterms are now evaluated

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\vdots \qquad \vdots}{\Xi}}{B}\,[r'_2] \qquad \Phi}{\Psi}\,[\textsc{Trans}]}{A}\,[r'_1] \qquad \Delta}{\Gamma}\,[\textsc{Trans}]$$

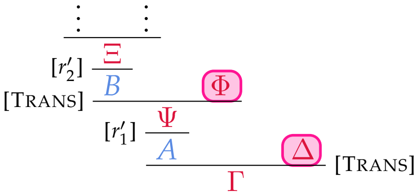
# 2. Specialized congruence rules

Subterms are now evaluated



# 2. Specialized congruence rules

Subterms are now evaluated



## 2. Specialized congruence rules

Subterms are now evaluated

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\vdots \qquad \vdots}{\Xi}}{B}\,[r_2'] \qquad \Phi}{\Psi}\,[\textsc{Trans}]}{A}\,[r_1'] \qquad \Delta}{\Gamma}\,[\textsc{Trans}]$$

But

- mutually inductive $\rightarrow$ and $\searrow$

# Refocusing in SOS

Small-step $\longrightarrow$ pretty-big-step SOS by

1. introduce refocusing rule
2. specialize congruence rules
3. specialize evaluation rules

# 3. Specialize evaluation rules

**Rules**

$$\frac{n = n_1 + n_2}{\textsf{plus}(n_1, n_2) \rightarrow n}\ [\textsc{Plus}]$$

$$\frac{t_1 \searrow v_1}{\textsf{plus}(t_1, e_2) \rightarrow \textsf{plus}(v_1, e_2)}\ [\textsc{Plus}1'] \qquad \frac{t_2 \searrow v_2}{\textsf{plus}(v_1, t_2) \rightarrow \textsf{plus}(v_1, v_2)}\ [\textsc{Plus}2']$$

Specialization by unfolding [Trans] wrt [Plus1′]

$$\frac{t \rightarrow e' \qquad e' \searrow v}{t \searrow v}\ [\textsc{Trans}]$$

# 3. Specialize evaluation rules

**Rules**

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \rightarrow n}\ [\textsc{Plus}]$$

$$\frac{t_1 \searrow v_1}{\mathsf{plus}(t_1, e_2) \rightarrow \mathsf{plus}(v_1, e_2)}\ [\textsc{Plus1}'] \qquad \frac{t_2 \searrow v_2}{\mathsf{plus}(v_1, t_2) \rightarrow \mathsf{plus}(v_1, v_2)}\ [\textsc{Plus2}']$$

Specialization by unfolding [TRANS] wrt [PLUS1′]

$$\frac{\dfrac{t_1 \searrow v_1}{\mathsf{plus}(t_1, e_2) \rightarrow \mathsf{plus}(v_1, e_2)}\ [\textsc{Plus1}'] \qquad \mathsf{plus}(v_1, e_2) \searrow v}{\mathsf{plus}(t_1, e_2) \searrow v}\ [\textsc{Trans}]$$

# 3. Specialize evaluation rules

**Rules**

$$\frac{n = n_1 + n_2}{\textsf{plus}(n_1, n_2) \rightarrow n}\ [\textsc{Plus}]$$

$$\frac{t_1 \searrow v_1}{\textsf{plus}(t_1, e_2) \rightarrow \textsf{plus}(v_1, e_2)}\ [\textsc{Plus1}'] \qquad \frac{t_2 \searrow v_2}{\textsf{plus}(v_1, t_2) \rightarrow \textsf{plus}(v_1, v_2)}\ [\textsc{Plus2}']$$

Specialization by unfolding [TRANS] wrt [PLUS1′]

$$\frac{t_1 \searrow v_1 \qquad \textsf{plus}(v_1, e_2) \searrow v}{\textsf{plus}(t_1, e_2) \searrow v}\ [\textsc{Trans-Plus1}']$$

# 3. Specialize evaluation rules

**Rules**

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \to n}\ [\textsc{Plus}]$$

$$\frac{t_1 \searrow v_1}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(v_1, e_2)}\ [\textsc{Plus1}']
\qquad
\frac{t_2 \searrow v_2}{\mathsf{plus}(v_1, t_2) \to \mathsf{plus}(v_1, v_2)}\ [\textsc{Plus2}']$$

Specialization by unfolding [TRANS] wrt [PLUS1′]

$$\frac{t_1 \Downarrow v_1 \qquad \mathsf{plus}(v_1, e_2) \Downarrow v}{\mathsf{plus}(t_1, e_2) \Downarrow v}\ [\textsc{PBPlus1}]$$

# 3. Specialize evaluation rules

**Rules**

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \to n}\ [\textsc{Plus}]$$

$$\frac{t_1 \searrow v_1}{\mathsf{plus}(t_1, e_2) \to \mathsf{plus}(v_1, e_2)}\ [\textsc{Plus1}'] \qquad \frac{t_2 \searrow v_2}{\mathsf{plus}(v_1, t_2) \to \mathsf{plus}(v_1, v_2)}\ [\textsc{Plus2}']$$

Specialization by unfolding [TRANS] wrt [PLUS1′]

$$\frac{t_1 \Downarrow v_1 \qquad \mathsf{plus}(v_1, e_2) \Downarrow v}{\mathsf{plus}(t_1, e_2) \Downarrow v}\ [\textsc{PBPlus1}]$$

Rules for $\to$ become redundant!

# 3. Specialized evaluation rules

## Language syntax

$$Expr \ni e ::= v \mid t$$

$$Val \ni v ::= n \in \mathbb{N} \qquad Term \ni t ::= \mathsf{plus}(e, e)$$

## Pretty-big-step rules

$$\boxed{e \Downarrow v}$$

$$\frac{}{v \Downarrow v}\ [\text{PBReflV}]$$

$$\frac{t_1 \Downarrow v_1 \qquad \mathsf{plus}(v_1, e_2) \Downarrow v}{\mathsf{plus}(t_1, e_2) \Downarrow v}\ [\text{PBPlus1}]$$

$$\frac{t_2 \Downarrow v_2 \qquad \mathsf{plus}(v_1, v_2) \Downarrow v}{\mathsf{plus}(v_1, t_2) \Downarrow v}\ [\text{PBPlus2}]$$

$$\frac{n = n_1 + n_2}{\mathsf{plus}(n_1, n_2) \Downarrow n}\ [\text{PBPlus}]$$

# 3. Specialized evaluation rules

## Language syntax

$$Expr \ni e ::= v \mid t \quad Val \ni v ::= n \in \mathbb{N} \quad Term \ni t ::= \mathsf{plus}(e, e)$$

$$AbrTer \ni a ::= \tau \mid \mathsf{exc} \qquad Outcome \ni o ::= \langle v, \tau \rangle \mid \langle e, \mathsf{exc} \rangle$$

## Pretty-big-step rules with abrupt termination

$$\frac{}{\langle v, \tau \rangle \Downarrow \langle v, \tau \rangle}\ [\text{PBReflV}] \qquad \frac{}{\langle e, \mathsf{exc} \rangle \Downarrow \langle e, \mathsf{exc} \rangle}\ [\text{PBReflE}] \qquad \boxed{\langle e, a \rangle \Downarrow o}$$

$$\frac{\langle t_1, \tau \rangle \Downarrow \langle e_1', a \rangle \qquad \langle \mathsf{plus}(e_1', e_2), a \rangle \Downarrow o}{\langle \mathsf{plus}(t_1, e_2), \tau \rangle \Downarrow o}\ [\text{PBPlus1}]$$

$$\frac{\langle t_2, \tau \rangle \Downarrow \langle e_2', a \rangle \qquad \langle \mathsf{plus}(v_1, e_2'), a \rangle \Downarrow o}{\langle \mathsf{plus}(v_1, t_2), \tau \rangle \Downarrow o}\ [\text{PBPlus2}]$$

$$\frac{n = n_1 + n_2}{\langle \mathsf{plus}(n_1, n_2), \tau \rangle \Downarrow \langle n, \tau \rangle}\ [\text{PBPlus}] \qquad \frac{}{\langle \mathsf{throw}, \tau \rangle \Downarrow \langle 0, \mathsf{exc} \rangle}\ [\text{Throw}]$$

# 3. Specialized evaluation rules

## Language syntax

$$Expr \ni e ::= v \mid t \qquad Val \ni v ::= n \in \mathbb{N} \qquad Term \ni t ::= \mathsf{plus}(e, e)$$

$$AbrTer \ni a ::= \tau \mid \mathsf{exc} \qquad Outcome \ni o ::= \langle v, \tau \rangle \mid \langle e, \mathsf{exc} \rangle$$

## Pretty-big-step rules with abrupt termination

$$\frac{}{\langle v, \tau \rangle \Downarrow \langle v, \tau \rangle}\ [\text{PBReflV}] \qquad \frac{}{\langle e, \mathsf{exc} \rangle \Downarrow \langle e, \mathsf{exc} \rangle}\ [\text{PBReflE}] \qquad \boxed{\langle e, a \rangle \Downarrow o}$$

$$\frac{\langle t_1, \tau \rangle \Downarrow \langle e_1', a \rangle \qquad \langle \mathsf{plus}(e_1', e_2), a \rangle \Downarrow o}{\langle \mathsf{plus}(t_1, e_2), \tau \rangle \Downarrow o}\ [\text{PBPlus1}]$$

$$\frac{\langle t_2, \tau \rangle \Downarrow \langle e_2', a \rangle \qquad \langle \mathsf{plus}(v_1, e_2'), a \rangle \Downarrow o}{\langle \mathsf{plus}(v_1, t_2), \tau \rangle \Downarrow o}\ [\text{PBPlus2}]$$

$$\frac{n = n_1 + n_2}{\langle \mathsf{plus}(n_1, n_2), \tau \rangle \Downarrow \langle n, \tau \rangle}\ [\text{PBPlus}] \qquad \frac{}{\langle \mathsf{throw}, \tau \rangle \Downarrow \langle 0, \mathsf{exc} \rangle}\ [\text{Throw}]$$

# 3. Specialized evaluation rules

## Language syntax

$$Expr \ni e ::= v \mid t \quad Val \ni v ::= n \in \mathbb{N} \quad Term \ni t ::= \mathsf{plus}(e, e)$$

$$AbrTer \ni a ::= \tau \mid \mathsf{exc} \qquad Outcome \ni o ::= \langle v, \tau \rangle \mid \langle e, \mathsf{exc} \rangle$$

## Pretty-big-step rules with abrupt termination

$$\frac{}{\langle v, \tau \rangle \Downarrow \langle v, \tau \rangle}\ [\text{PBReflV}] \qquad \frac{}{\langle e, \mathsf{exc} \rangle \Downarrow \langle e, \mathsf{exc} \rangle}\ [\text{PBReflE}] \qquad \boxed{\langle e, a \rangle \Downarrow o}$$

$$\frac{\langle t_1, \tau \rangle \Downarrow \langle e'_1, a \rangle \qquad \langle \mathsf{plus}(e'_1, e_2), a \rangle \Downarrow o}{\langle \mathsf{plus}(t_1, e_2), \tau \rangle \Downarrow o}\ [\text{PBPlus1}]$$

$$\frac{\langle t_2, \tau \rangle \Downarrow \langle e'_2, a \rangle \qquad \langle \mathsf{plus}(v_1, e'_2), a \rangle \Downarrow o}{\langle \mathsf{plus}(v_1, t_2), \tau \rangle \Downarrow o}\ [\text{PBPlus2}]$$

$$\frac{n = n_1 + n_2}{\langle \mathsf{plus}(n_1, n_2), \tau \rangle \Downarrow \langle n, \tau \rangle}\ [\text{PBPlus}] \qquad \frac{}{\langle \mathsf{throw}, \tau \rangle \Downarrow \langle 0, \mathsf{exc} \rangle}\ [\text{Throw}]$$

# 3. Specialized evaluation rules

## Language syntax

$$Expr \ni e ::= v \mid t \qquad Val \ni v ::= n \in \mathbb{N} \qquad Term \ni t ::= \mathsf{plus}(e, e)$$

$$AbrTer \ni a ::= \tau \mid \mathsf{exc} \qquad Outcome \ni o ::= \langle v, \tau \rangle \mid \langle e, \mathsf{exc} \rangle$$

## Pretty-big-step rules with abrupt termination

$$\frac{}{\langle v, \tau \rangle \Downarrow \langle v, \tau \rangle}\ [\text{PBReflV}] \qquad \frac{}{\langle e, \mathsf{exc} \rangle \Downarrow \langle e, \mathsf{exc} \rangle}\ [\text{PBReflE}] \qquad \boxed{\langle e, a \rangle \Downarrow o}$$

$$\frac{\langle t_1, \tau \rangle \Downarrow \langle e_1', a \rangle \qquad \langle \mathsf{plus}(e_1', e_2), a \rangle \Downarrow o}{\langle \mathsf{plus}(t_1, e_2), \tau \rangle \Downarrow o}\ [\text{PBPlus1}]$$

$$\frac{\langle t_2, \tau \rangle \Downarrow \langle e_2', a \rangle \qquad \langle \mathsf{plus}(v_1, e_2'), a \rangle \Downarrow o}{\langle \mathsf{plus}(v_1, t_2), \tau \rangle \Downarrow o}\ [\text{PBPlus2}]$$

$$\frac{n = n_1 + n_2}{\langle \mathsf{plus}(n_1, n_2), \tau \rangle \Downarrow \langle n, \tau \rangle}\ [\text{PBPlus}] \qquad \frac{}{\langle \mathsf{throw}, \tau \rangle \Downarrow \langle 0, \mathsf{exc} \rangle}\ [\text{Throw}]$$

# 3. Specialized evaluation rules

## Language syntax

$$Expr \ni e ::= v \mid t \quad Val \ni v ::= n \in \mathbb{N} \quad Term \ni t ::= \mathsf{plus}(e, e)$$

$$AbrTer \ni a ::= \boldsymbol{\tau} \mid \mathsf{exc} \qquad Outcome \ni o ::= \langle v, \boldsymbol{\tau} \rangle \mid \langle e, \mathsf{exc} \rangle$$

## Pretty-big-step rules with abrupt termination

$$\frac{}{\langle v, \boldsymbol{\tau} \rangle \Downarrow \langle v, \boldsymbol{\tau} \rangle}\ [\text{PBReflV}] \qquad \frac{}{\langle e, \mathsf{exc} \rangle \Downarrow \langle e, \mathsf{exc} \rangle}\ [\text{PBReflE}] \qquad \boxed{\langle e, a \rangle \Downarrow o}$$

$$\frac{\langle t_1, \boldsymbol{\tau} \rangle \Downarrow \langle e_1', a \rangle \qquad \langle \mathsf{plus}(e_1', e_2), a \rangle \Downarrow \boxed{o}}{\langle \mathsf{plus}(t_1, e_2), \boldsymbol{\tau} \rangle \Downarrow \boxed{o}}\ [\text{PBPlus1}]$$

$$\frac{\langle t_2, \boldsymbol{\tau} \rangle \Downarrow \langle e_2', a \rangle \qquad \langle \mathsf{plus}(v_1, e_2'), a \rangle \Downarrow o}{\langle \mathsf{plus}(v_1, t_2), \boldsymbol{\tau} \rangle \Downarrow o}\ [\text{PBPlus2}]$$

$$\frac{n = n_1 + n_2}{\langle \mathsf{plus}(n_1, n_2), \boldsymbol{\tau} \rangle \Downarrow \langle n, \boldsymbol{\tau} \rangle}\ [\text{PBPlus}] \qquad \frac{}{\langle \mathsf{throw}, \boldsymbol{\tau} \rangle \Downarrow \langle 0, \mathsf{exc} \rangle}\ [\text{Throw}]$$

# Correctness

proof method, criteria, and Coq mechanisation

`http://plancomps.org/bachpoulsen2013a/coq`

# Soundness of derived pretty-big-step relation ($\Downarrow$)

**Soundness theorem**

$e \Downarrow v$ implies $e \rightarrow^* v$.

Proof. By structural rule induction, transitivity of $\rightarrow^*$, and congruence lemmas [Leroy and Grall, '09].

# Soundness of derived pretty-big-step relation ($\Downarrow$)

**Soundness theorem**

$e \Downarrow v$ implies $e \rightarrow^* v$.

*Proof.* By structural rule induction, transitivity of $\rightarrow^*$, and congruence lemmas [Leroy and Grall, '09].

**Reflexive-transitive correspondence lemma**

$e \searrow v$ iff $e \rightarrow^* v$.

# Soundness of derived pretty-big-step relation ($\Downarrow$)

**Soundness theorem**

$e \Downarrow v$ implies $e \rightarrow^* v$.

Proof. By structural rule induction, transitivity of $\rightarrow^*$, and congruence lemmas [Leroy and Grall, '09].

**Reflexive-transitive correspondence lemma**

$e \searrow v$ iff $e \rightarrow^* v$.

**Congruence lemma (example)**

$e_1 \rightarrow^* e_1'$ implies $\mathsf{plus}(e_1, e_2) \rightarrow^* \mathsf{plus}(e_1', e_2)$.

# Soundness of derived pretty-big-step relation ($\Downarrow$)

**Soundness theorem**

$e \Downarrow v$ implies $e \rightarrow^* v$.

Proof. By structural rule induction, transitivity of $\rightarrow^*$, and congruence lemmas [Leroy and Grall, '09].

**Reflexive-transitive correspondence lemma**

$e \searrow v$ iff $e \rightarrow^* v$.

**Congruence lemma (example)**

$e_1 \rightarrow^* e_1'$ implies $\mathsf{plus}(e_1, e_2) \rightarrow^* \mathsf{plus}(e_1', e_2)$.

Observation: refocusing is sound for compositional rules.

# Completeness of derived pretty-big-step relation ($\Downarrow$)

**Completeness theorem**

$e \rightarrow^* v$ implies $e \Downarrow v$.

Proof. By structural rule induction and big-step decomposition [Leroy and Grall, '09].

# Completeness of derived pretty-big-step relation ($\Downarrow$)

**Completeness theorem**

$e \to^* v$ implies $e \Downarrow v$.

Proof. By structural rule induction and big-step decomposition [Leroy and Grall, '09].

**Big-step decomposition lemma**

$e \to e'$ and $e' \Downarrow v$ implies $e \Downarrow v$.

# Completeness of derived pretty-big-step relation ($\Downarrow$)

## Completeness theorem

$e \rightarrow^* v$ implies $e \Downarrow v$.

Proof. By structural rule induction and big-step decomposition [Leroy and Grall, '09].

## Big-step decomposition lemma

$e \rightarrow e'$ and $e' \Downarrow v$ implies $e \Downarrow v$.

Observation: refocusing is complete for deterministic semantics.

# Scaling to other language features

Refocusing an ML-like language with

- applicative environment
- imperative store
- abrupt termination

# Scaling to other language features

Refocusing an ML-like language with

- applicative environment
- imperative store
- abrupt termination

Correctness:

- ✓ deterministic
- non-compositional catch rule;

# Scaling to other language features

Refocusing an ML-like language with

- applicative environment
- imperative store
- abrupt termination

Correctness:

✓ deterministic

✓ non-compositional `catch` rule; required auxiliary similarity lemmas; see

`http://plancomps.org/bachpoulsen2014a/coq`

# Conclusion and further directions

In this talk:

- Small-step vs. big-step? We don't have to choose!
  - pretty-big-step rules are automatically derivable from small-step rules
- Correctness:
  - Soundness: compositionality
  - Completeness: determinism

Further directions: PLanCompS (`http://plancomps.org`)

- component-based semantics
- verification (e.g., type soundness)
- validation (e.g., prototype interpreters)